Mobile Communications
TCS 455

Dr. Prapun Suksompong
prapun@siit.tu.ac.th

Lecture 12

Office Hours:
BKD 3601-7
Tuesday 14:00-16:00
Thursday 9:30-11:30

Announcements

- Read
  - Chapter 3: 3.1 – 3.2, 3.5.1, 3.6, 3.7.2
    - Posted on the web
  - Appendix A.1 (Erlang B)
- Due date for HW3: Dec 18

Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

"Capacity" $C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$

Tradeoff

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

- Omni-directional: K = 6
- 120° Sectoring: K = 2
- 60° Sectoring: K = 1

Trunking

Call blocking probability

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Erlang-B formula

λ = Average # call attempts/requests per unit time

$A =$ traffic intensity or load [Erlangs] $= \frac{\lambda}{\mu}$

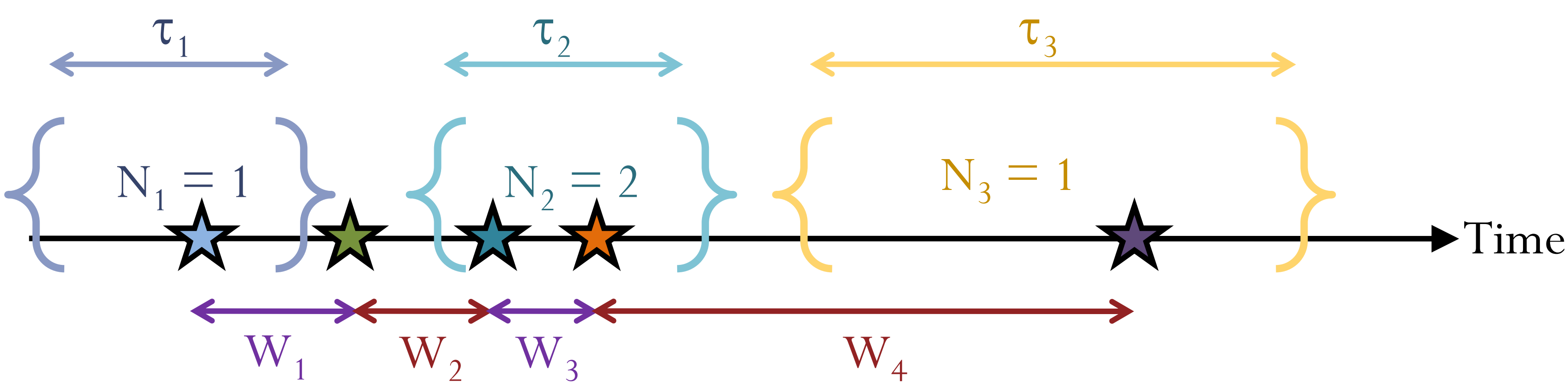
$\frac{1}{\mu} = H =$ Average call length

Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration of the time that a user occupies a channel is exponentially distributed**, so that longer calls are less likely to occur.
- There are m channels available in the trunking pool.
  - For us, m = the number of channels for a cell (C) or for a sector

Poisson Process

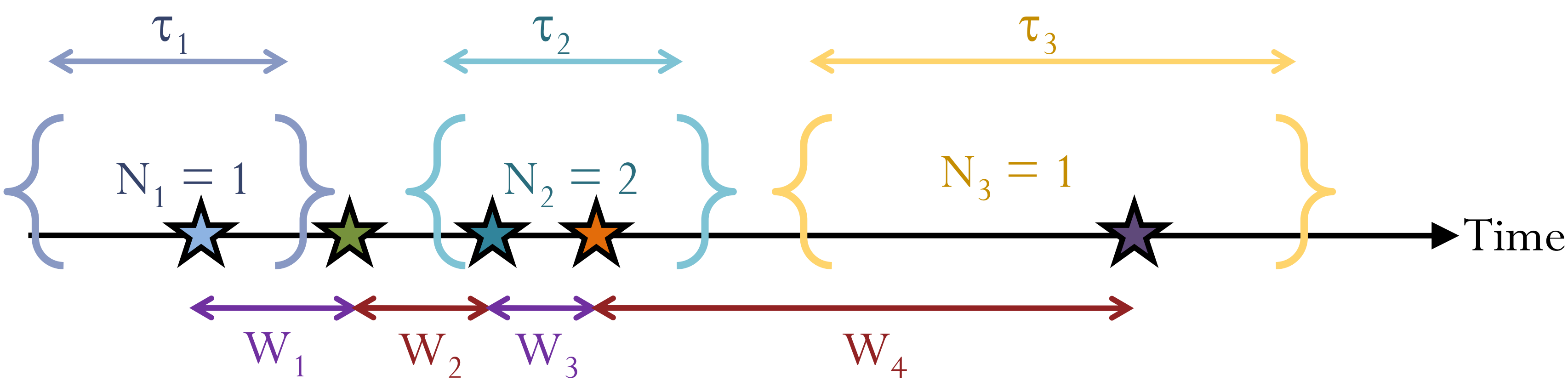
The number of arrivals N_1, N_2 and N_3 during non-overlapping time intervals are independent Poisson random variables with mean = $\lambda \times$ the length of the corresponding interval.



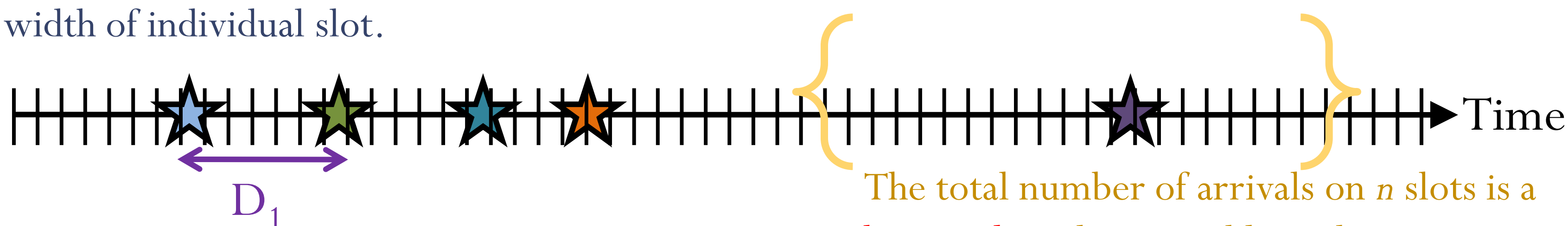
The lengths of time between adjacent arrivals W_1, W_2, W_3 … are i.i.d. exponential random variables with mean $1/\lambda$.

Small Slot Analysis (Poisson Process)

- Aka discrete time approximation



In the limit, there is at most one arrival in any slot. The numbers of arrivals on the slots are i.i.d. Bernoulli random variables with probability p_1 of exactly one arrivals $= \lambda\delta$ where δ is the width of individual slot.



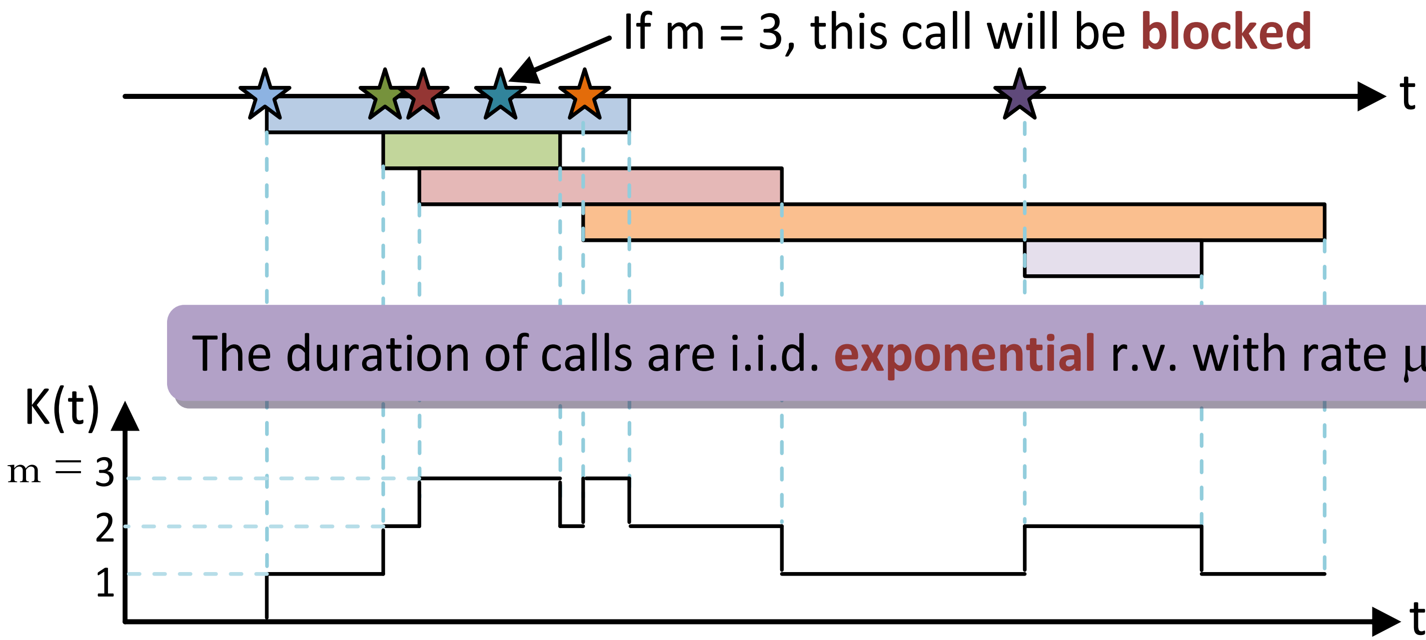
The number of slots between adjacent arrivals is a geometric random variable.

The total number of arrivals on *n* slots is a binomial random variable with parameter (n, p_1)

In the limit, as the slot length gets smaller, geometric $\longrightarrow$ exponential
binomial $\longrightarrow$ Poisson

Assumption (2)

The call request process is **Poisson** with rate λ



The duration of calls are i.i.d. **exponential** r.v. with rate μ.

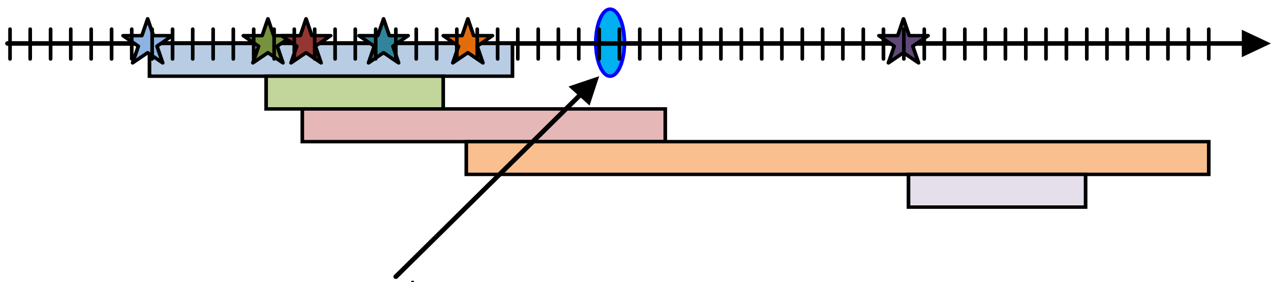
K(t) = "state" of the system

= the number of used channel at time *t*

We want to find out what proportion of time the system has $K = m$.

Small Slot Analysis (Erlang B)

Suppose each slot duration is δ.



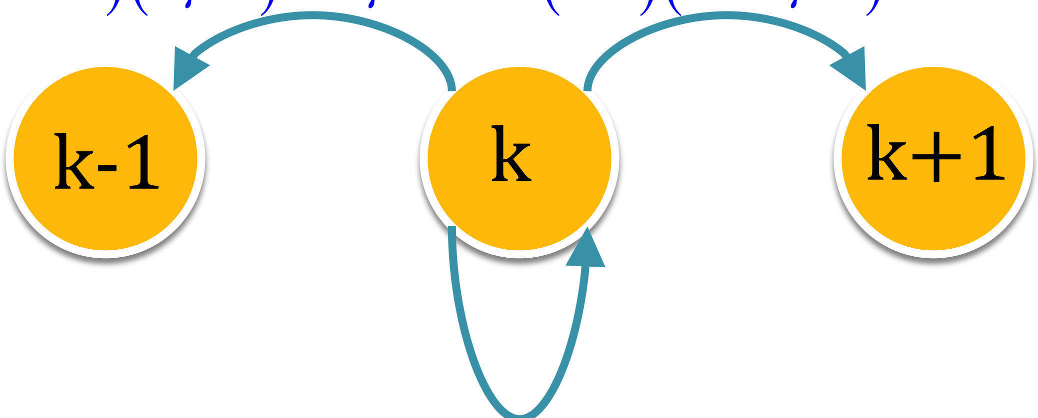
- Consider the i^{th} small slot.
- Let K_i = k be the value of K at the beginning of this time slot.
- k = 2 in the above figure.
- Then, K_{i+1} is the value of K at the end of this slot which is the same as the value of K at the beginning of the next slot.
- P[0 new call request] ≈ $1 - \lambda\delta$
- P[1 new call request] ≈ $\lambda\delta$
- P[0 old-call end] ≈ $(1-\mu\delta)^k \approx 1 - k\mu\delta$
- P[1 old-call end] ≈ $k\mu\delta(1-\mu\delta)^{k-1} \approx k\mu\delta$

How do these events affect K_{i+1}?

Small slot Analysis (2)

$$K_{i+1} = K_i + (\#\text{ new call request}) - (\#\text{ old-call end})$$

$$(1-\lambda\delta)(k\mu\delta) \approx k\mu\delta \qquad (\lambda\delta)(1-k\mu\delta) \approx \lambda\delta$$



$$(1-\lambda\delta)(1-k\mu\delta)+(\lambda\delta)(k\mu\delta) \approx 1-\lambda\delta-k\mu\delta$$

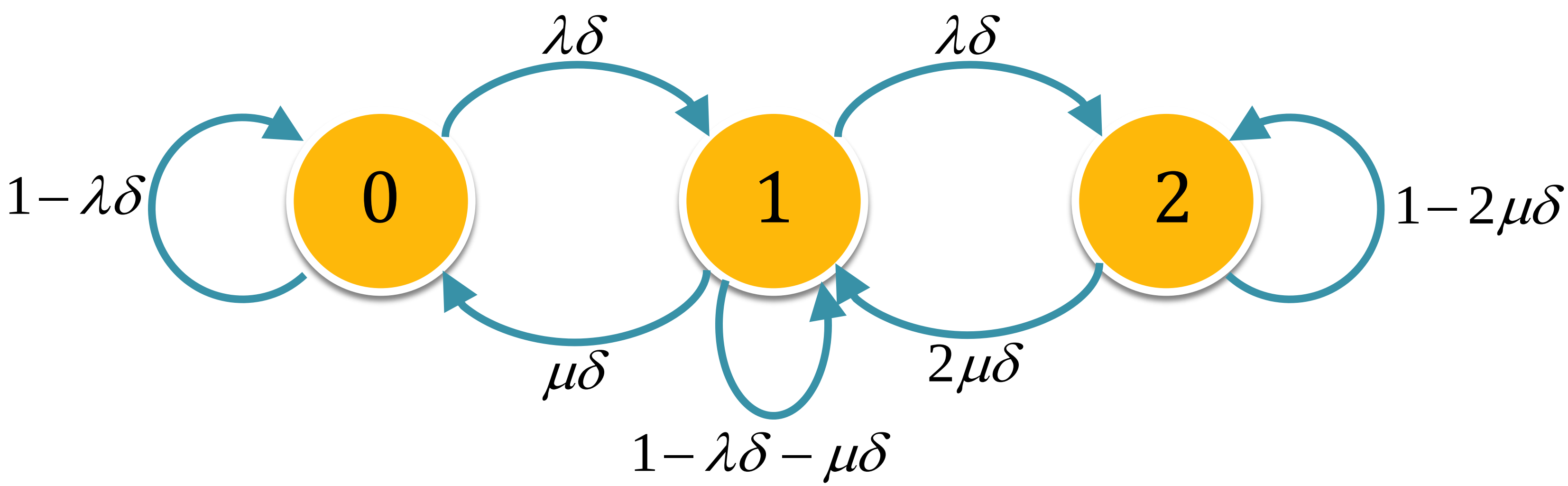
The labels on the arrows are probabilities.

P[0 new call request] ≈ 1 - $\lambda\delta$
P[1 new call request] ≈ $\lambda\delta$

P[0 old-call end] ≈ 1 - $k\mu\delta$
P[1 old-call end] ≈ $k\mu\delta$

Small slot Analysis: Markov Chain

- Case: m = 2



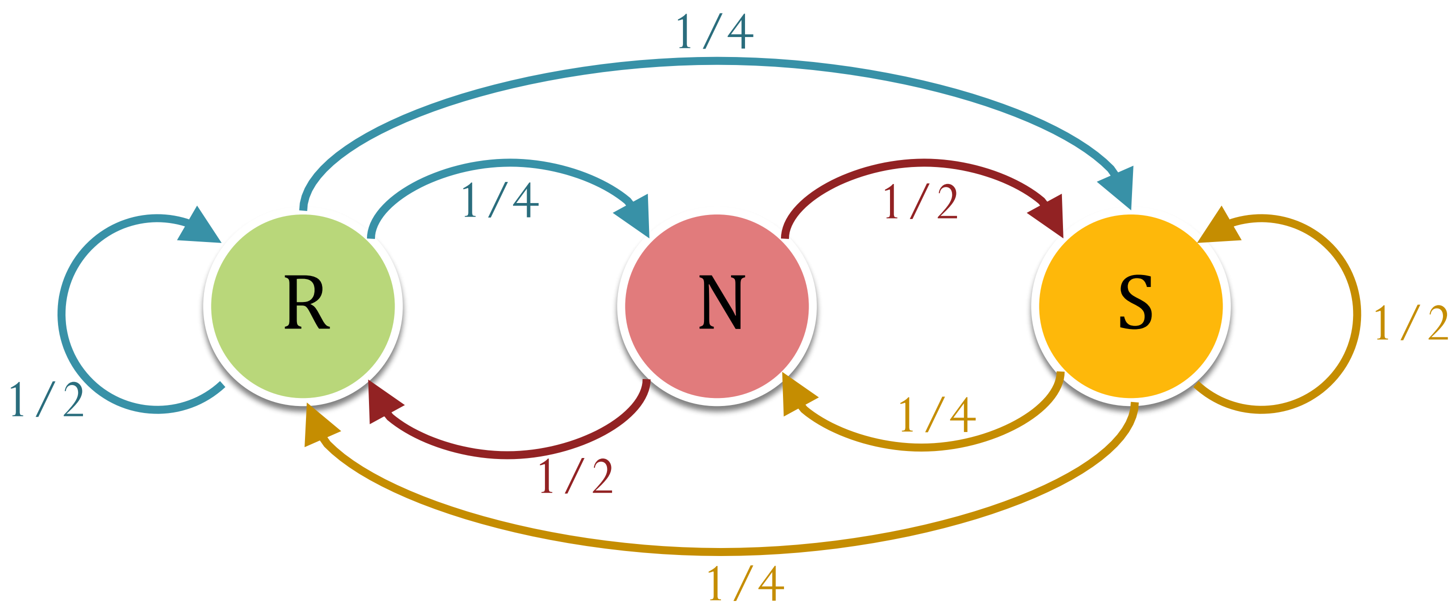
Markov Chain

- Markov chains model many phenomena of interest.
- We will see one important property: **Memoryless**
  - It retains no memory of where it has been in the past.
  - Only the current state of the process can influence where it goes next.
- Very similar to the *state transition diagram* in digital circuits.
  - In digital circuit, the labels on the arrows indicate the input/control signal.
  - Here, the labels on the arrows indicate transition probabilities. (If the system is currently at a particular state, where would it go next on the next time slot?)
- We will focus on **discrete time Markov chain**.

Example: The Land of Oz

- Land of Oz is blessed by many things, but not by good weather.
  - They never have two nice days in a row.
  - If they have a nice day, they are just as likely to have snow as rain the next day.
  - If they have snow or rain, they have an even chance of having the same the next day.
  - If there is change from snow or rain, only half of the time is this a change to a nice day.
- If you visit the land of Oz next year for one day, what is the chance that it will be a nice day?

State Transition Diagram



R = Rain
N = Nice
S = Snow

Markov Chain (2)

- Let K_i be the weather status for the i^{th} day (from today).
- Suppose we know that it is snowing in the land of Oz today. Then

$$K_0 = \text{S}$$

  where S means snow.
- **Goal**: We want to know whether $K_{365} = \text{N}$ where N means nice.
- Of course, the weather are controlled probabilistically; so we can only find $\mathbf{P}[K_{365} = \text{N}]$.
- From the specification (or from the state transition diagram), we know that

$$P[K_1 = \text{R}] = \frac{1}{4}, \quad P[K_1 = \text{N}] = \frac{1}{4}, \quad P[K_1 = \text{S}] = \frac{1}{2}$$

- Define vector

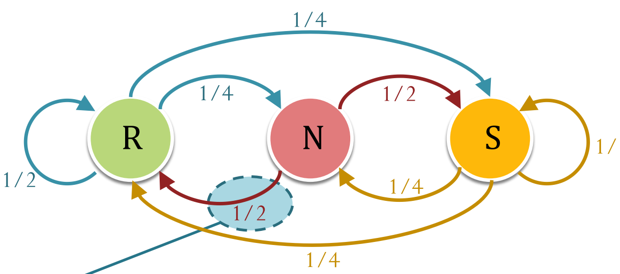
$$\vec{p}(i) = \begin{bmatrix} P[K_i = \text{R}] & P[K_i = \text{N}] & P[K_i = \text{S}] \end{bmatrix}$$

- Then,

$$\vec{p}(0) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \text{ and } \vec{p}(1) = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

The Land of Oz: Transition Matrix

$$\vec{p}(i+1) = \vec{p}(i) \times P$$



$$P\left[K_{i+1} = \text{rain} \middle| K_i = \text{normal}\right]$$

$$P = \begin{array}{c} \\ R \\ N \\ S \end{array} \begin{array}{c} \begin{array}{ccc} R & N & S \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \end{array}$$

$$\vec{p}(n) = \vec{p}(0) \times P^n$$

$$\vec{p}(2) = [0.3750 \quad 0.1875 \quad 0.4375]$$

$$\vec{p}(3) = [0.3906 \quad 0.2031 \quad 0.4063]$$

$$\vec{p}(5) = [0.3994 \quad 0.2002 \quad 0.4004]$$

$$\vec{p}(7) = [0.4000 \quad 0.2000 \quad 0.4000] = \vec{p}(8) = \vec{p}(9) = \vec{p}(10) = \cdots = \vec{p}(365)$$

Finding P^n for "large" n

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.4375 & 0.1875 & 0.3750 \\ 0.3750 & 0.2500 & 0.3750 \\ 0.3750 & 0.1875 & 0.4375 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.4063 & 0.2031 & 0.3906 \\ 0.4063 & 0.1875 & 0.4063 \\ 0.3906 & 0.2031 & 0.4063 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.4004 & 0.2002 & 0.3994 \\ 0.4004 & 0.1992 & 0.4004 \\ 0.3994 & 0.2002 & 0.4004 \end{bmatrix}$$

$$P^7 = \begin{bmatrix} 0.4000 & 0.2000 & 0.4000 \\ 0.4000 & 0.2000 & 0.4000 \\ 0.4000 & 0.2000 & 0.4000 \end{bmatrix} = P^8 = P^9 = P^{10} = \cdots$$

Land of Oz: Answer

- Recall that

$$\vec{p}(n) = \vec{p}(0) \times P^n$$

- So,

$$\vec{p}(7) = \vec{p}(0)P^7 = [0.4 \quad 0.2 \quad 0.4]$$

- Note that the above result is true regardless of the initial $\vec{p}(0)$
- $\vec{p}(365) = \vec{p}(0)P^{365} = [0.4 \quad 0.2 \quad 0.4]$

$\mathbf{P}[K_{365} = \mathbf{N}]$

Tip: Alternative look

$$P = \begin{bmatrix} 2/5 & 3/5 \\ 1/2 & 1/2 \end{bmatrix}$$